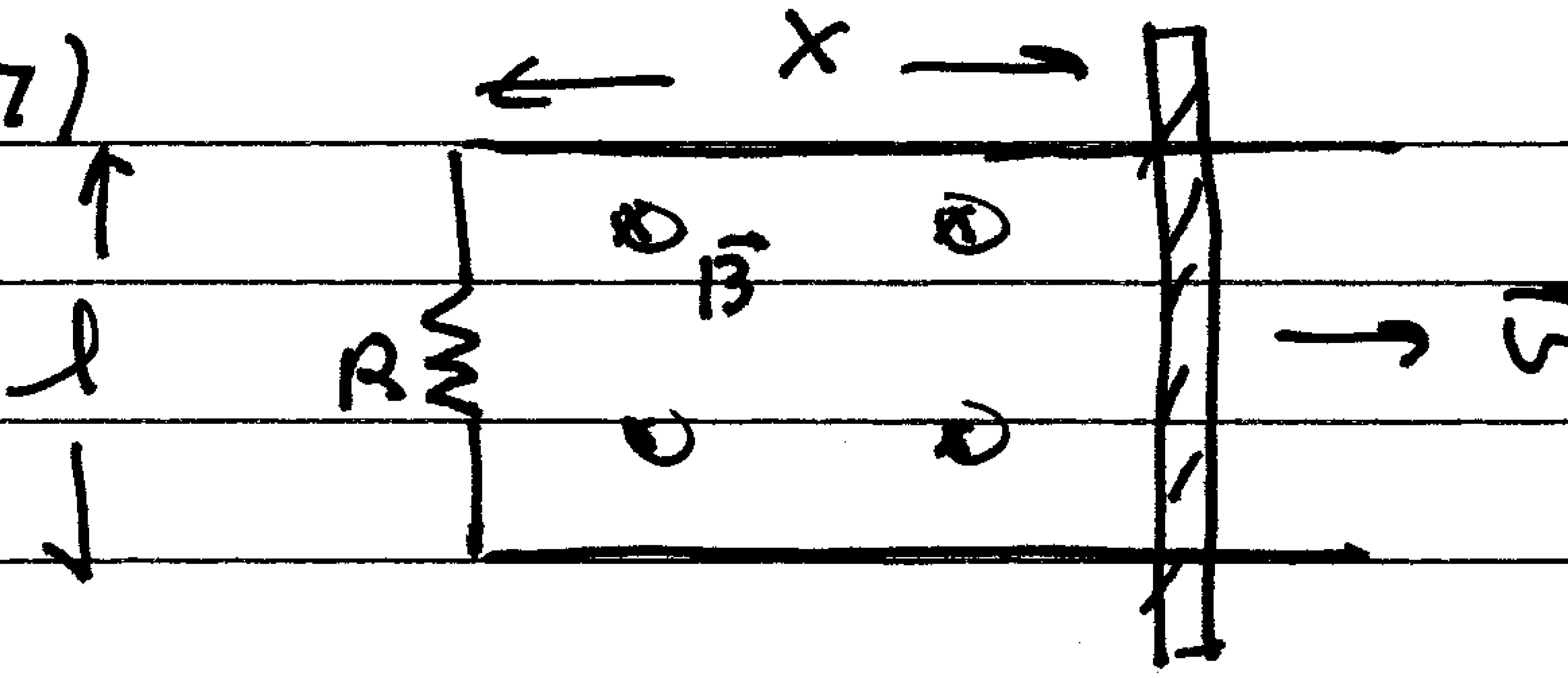
PHYS 321
Assignment 6
Due Wednesday, May 2, 2018

Read Chapter 7

Problems of Chapter 7: 7, 8, 16, 17, 18, 22, 33, 36, 55

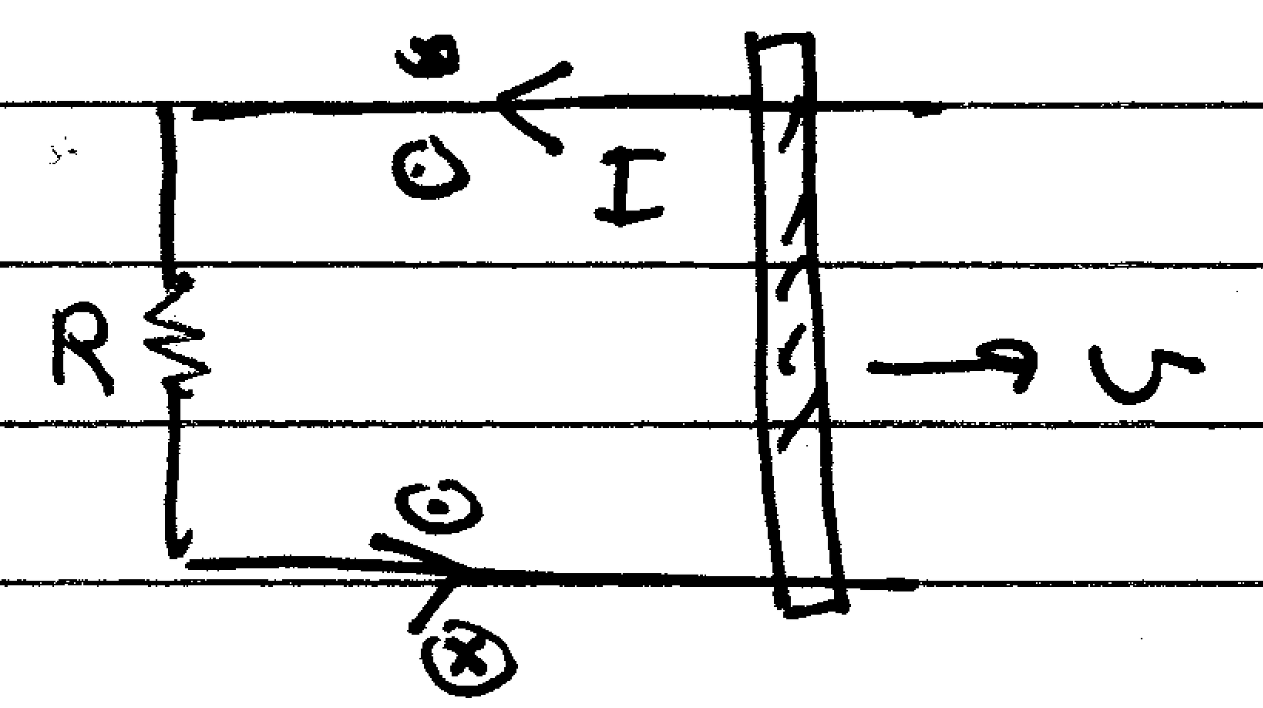
7-7)



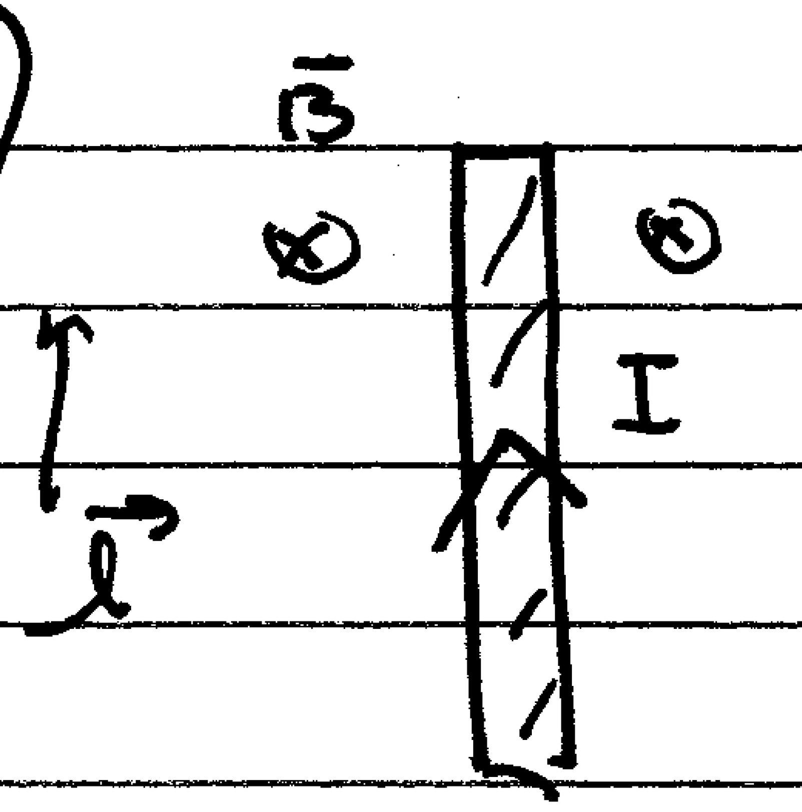
a) $|\mathcal{E}| = \left|\dfrac{d\phi_B}{dt}\right| \qquad \phi_B = x\,l\,B \qquad \dfrac{d\phi_B}{dt} = v\,l\,B$

$$|\mathcal{E}| = IR = v\,l\,B \qquad I = \frac{v\,l\,B}{R}$$

Since flux is increasing (as measured into the page) the current flows counter-clockwise



b)



$\vec{F} = I\,\vec{l} \times \vec{B} \qquad F = I\,l\,B \leftarrow$

$$F = \left(\frac{v\,l\,B}{R}\right)(lB) = \frac{v\,(lB)^2}{R} \leftarrow$$

c) $m\dfrac{dv}{dt} = F = -\dfrac{v\,(lB)^2}{R}$

$$\frac{dv}{dt} + \frac{(lB)^2}{mR}v = 0 \qquad v(t) = v_0 e^{-t/T}$$

$$T = \frac{mR}{(lB)^2}$$

d) The initial K.E. is $K = \frac{1}{2} m v_0^2$

$$\frac{dK}{dt} = - I^2 R = - \left(\frac{v \ell B}{R}\right)^2 R$$

$$= - \frac{(\ell B)^2}{R} v_0^2 e^{-2t/\tau}$$

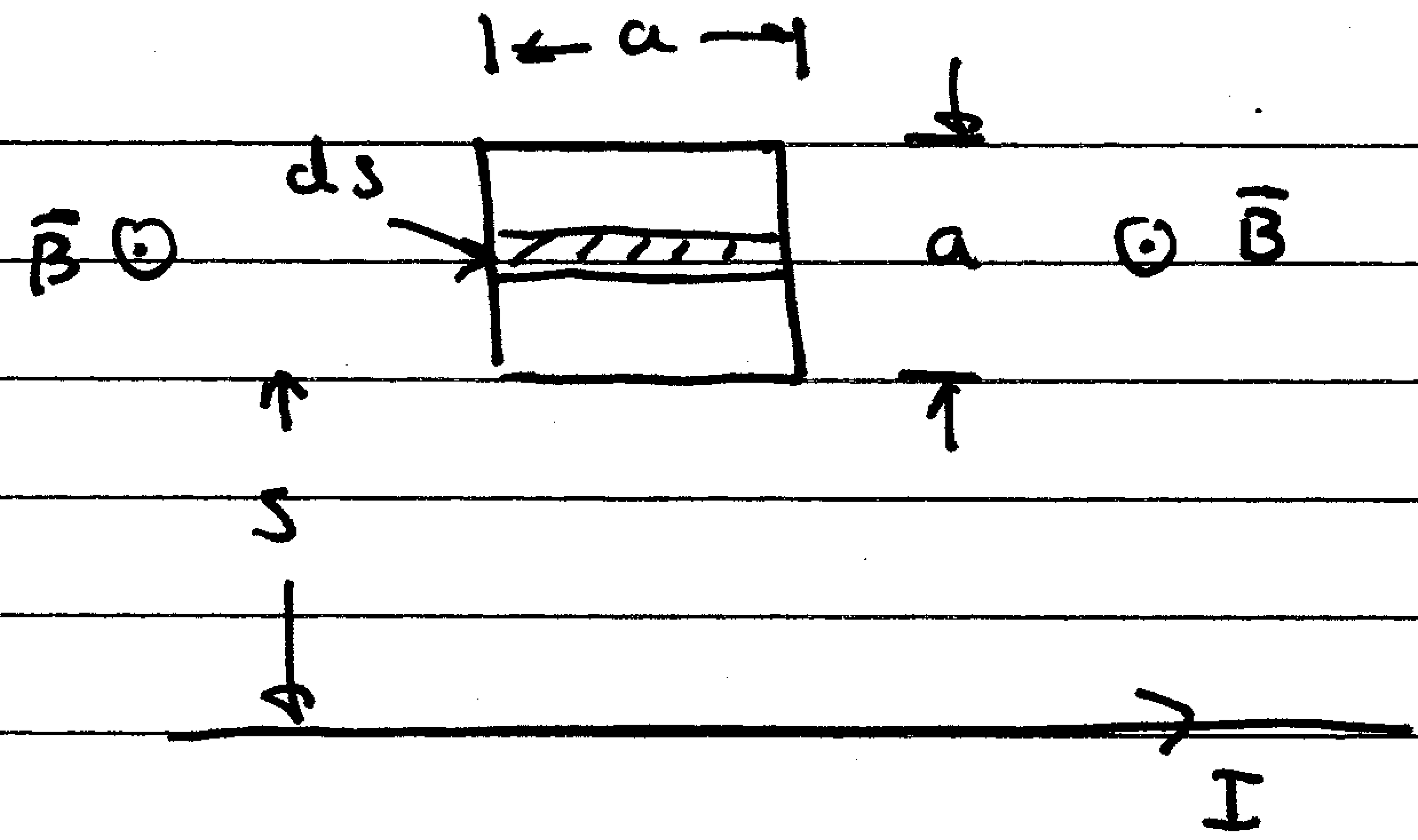
$$\int_0^\infty dt \frac{dK}{dt} = - \frac{(\ell B)^2 v_0^2}{R} \int_0^\infty dt\, e^{-2t/\tau}$$

$$\Delta K = - \frac{(\ell B)^2 v_0^2}{R} e^{-2t/\tau} \left(-\frac{\tau}{2}\right) \Bigg|_{t=0}^{t \to \infty}$$

$$\Delta K = - \frac{(\ell B)^2 v_0^2}{R} \frac{\tau}{2} = - \frac{(\ell B)^2 v_0^2}{2R} \frac{mR}{(\ell B)^2}$$

$$\Delta K = - \frac{1}{2} m v_0^2$$

7.8)



a) $B = \dfrac{\mu_0 I}{2\pi s}$ $\quad \phi = \int \vec{B} \cdot d\hat{a}$

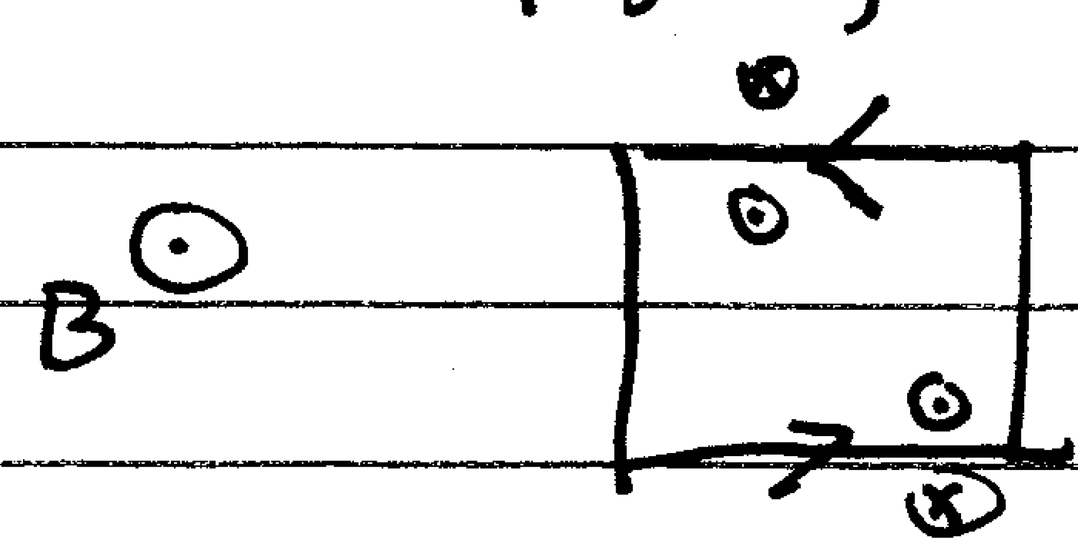
$$d\phi = \frac{\mu_0 I}{2\pi s}(a\,ds) \qquad \phi = \frac{\mu_0 I a}{2\pi} \int_s^{s+a} \frac{ds'}{s'}$$

$$\phi = \frac{\mu_0 I a}{2\pi} \ln\left(\frac{s+a}{s}\right)$$

b) $\dfrac{d\phi}{dt} = \dfrac{\mu_0 I a}{2\pi} \dfrac{d}{dt}\Big(\ln(s+a) - \ln s\Big)$ Let $\dfrac{ds}{dt} = v$

$$\frac{d\phi}{dt} = \frac{\mu_0 I a}{2\pi}\left(\frac{v}{s+a} - \frac{v}{s}\right) = -\frac{\mu_0 I a^2 v}{2\pi}\left(\frac{1}{s(s+a)}\right)$$

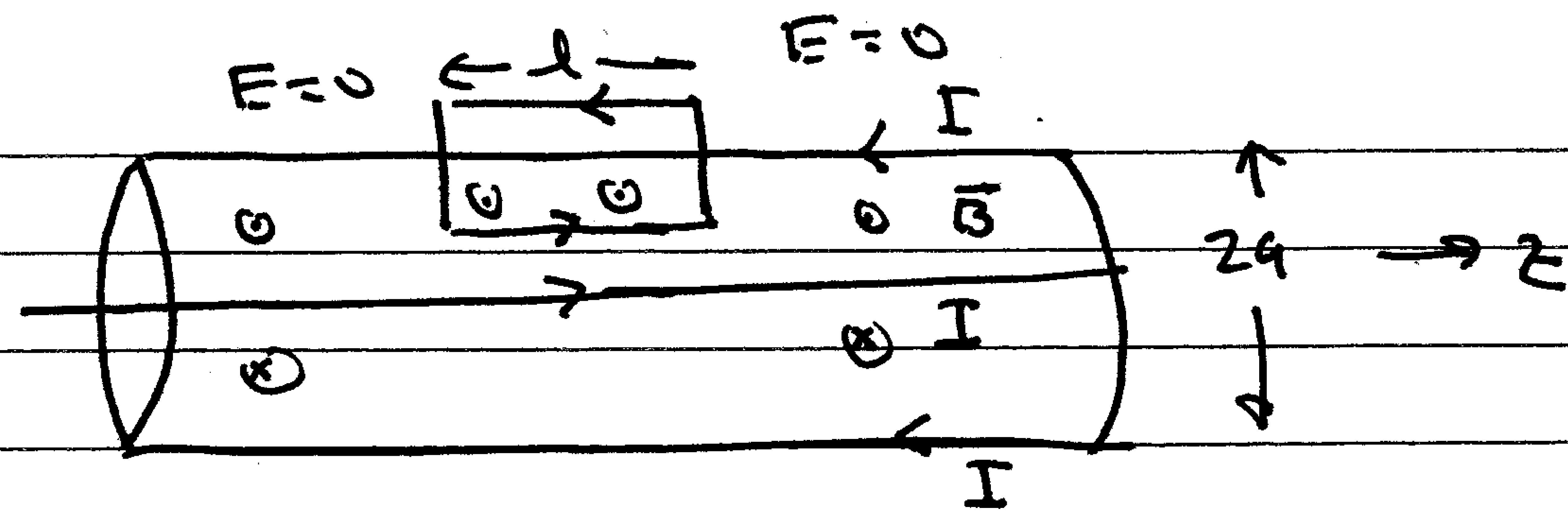
Since ϕ is decreasing (as measured out of the page), I flows to increase the ϕ.



counterclockwise flow

c) $\dfrac{d\phi}{dt} = 0 \Rightarrow \varepsilon = 0$

7-16)



$$B = \frac{\mu_0 I}{2\pi s} \quad a < s < 0 \qquad B = 0 \quad s > a$$

$$\oint \bar{E} \cdot d\bar{l} = -\frac{d\phi_B}{dt}$$ Consider loop integral drawn on diagram

$$\phi_B = \frac{\mu_0 I}{2\pi} \int_s^a \frac{ds'}{s'} = \frac{\mu_0 I l}{2\pi} \ln\left(\frac{a}{s}\right)$$

$$\oint \bar{E} \cdot d\bar{l} = E(s)\, l = -\frac{d\phi_B}{dt}$$

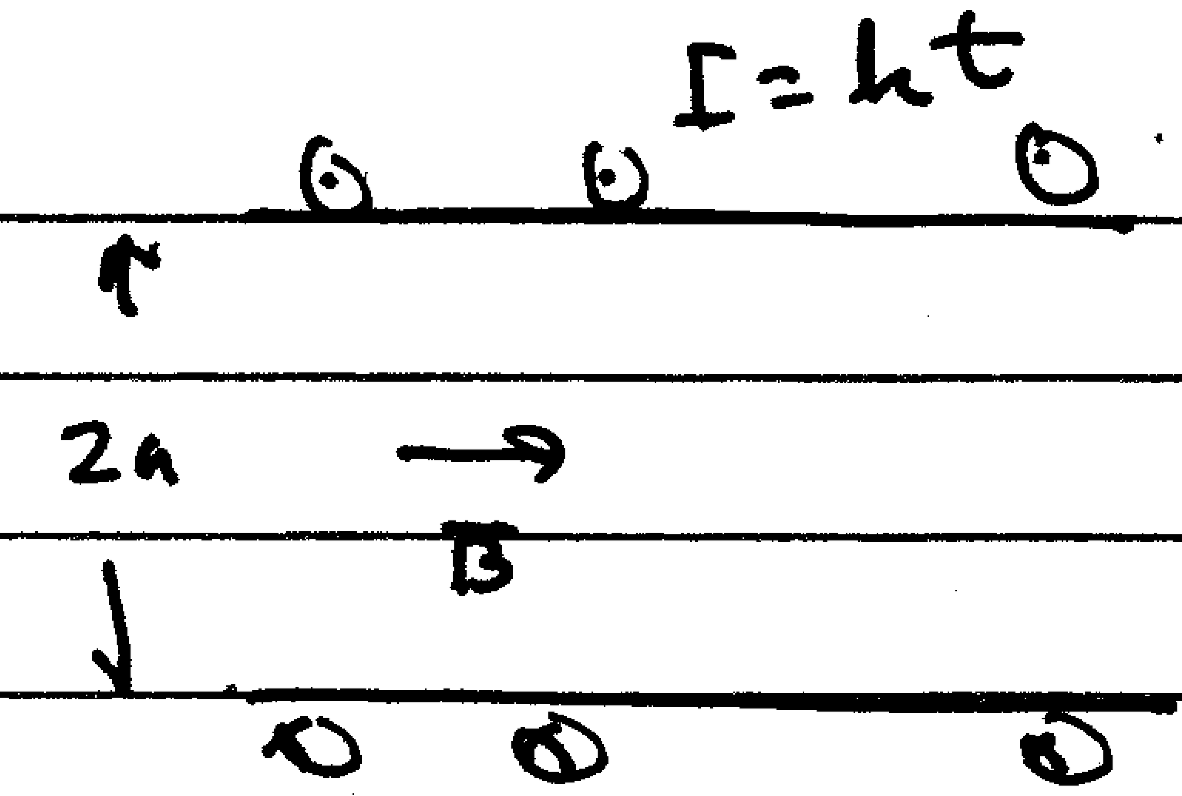
$$E(s)\, l = -\frac{d}{dt} \frac{\mu_0 I l}{2\pi} \ln\left(\frac{a}{s}\right)$$

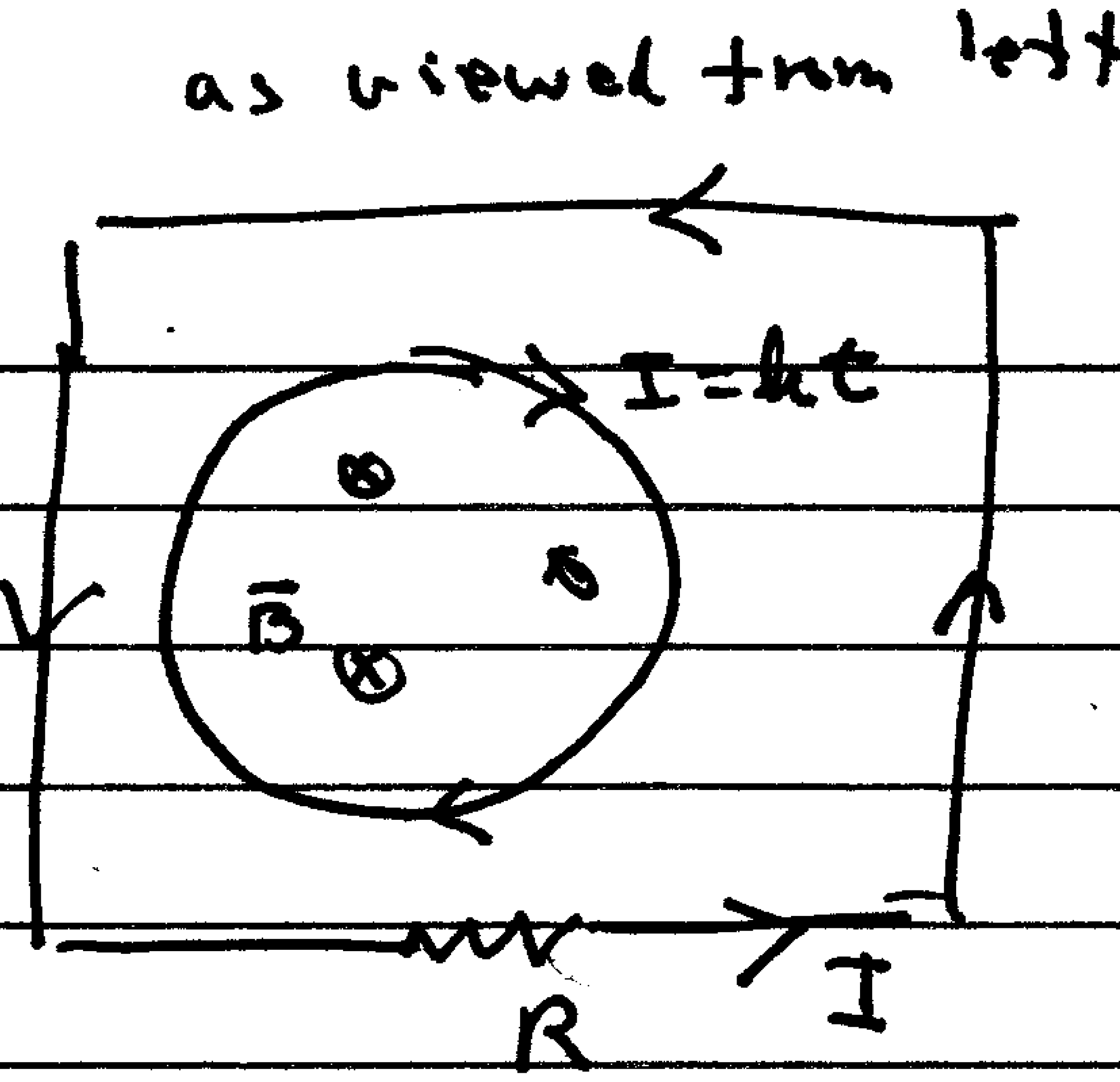
$$E(s) = -\frac{\mu_0}{2\pi} \frac{dI}{dt} \ln\left(\frac{a}{s}\right)$$

$$I(t) = I_0 \cos\omega t \qquad \frac{dI}{dt} = -I_0 \omega \sin\omega t$$

$$\vec{E}(s) = \frac{\mu_0 I_0 \omega}{2\pi} (\sin\omega t)\left(\ln \frac{a}{s}\right) \hat{z}$$

7-17)





$$\phi = B\pi a^2 = \mu_0 n I(\pi a^2)$$

$$\frac{d\phi}{dt} = \mu_0 \pi a^2 n \frac{dI}{dt} = \mathcal{E} = I_R R$$

$$I_R = \frac{\mu_0 \pi a^2 n k}{R} \qquad k \equiv \frac{dI}{dt}$$

Since ϕ is increasing, the current through the resistor R is counterclockwise

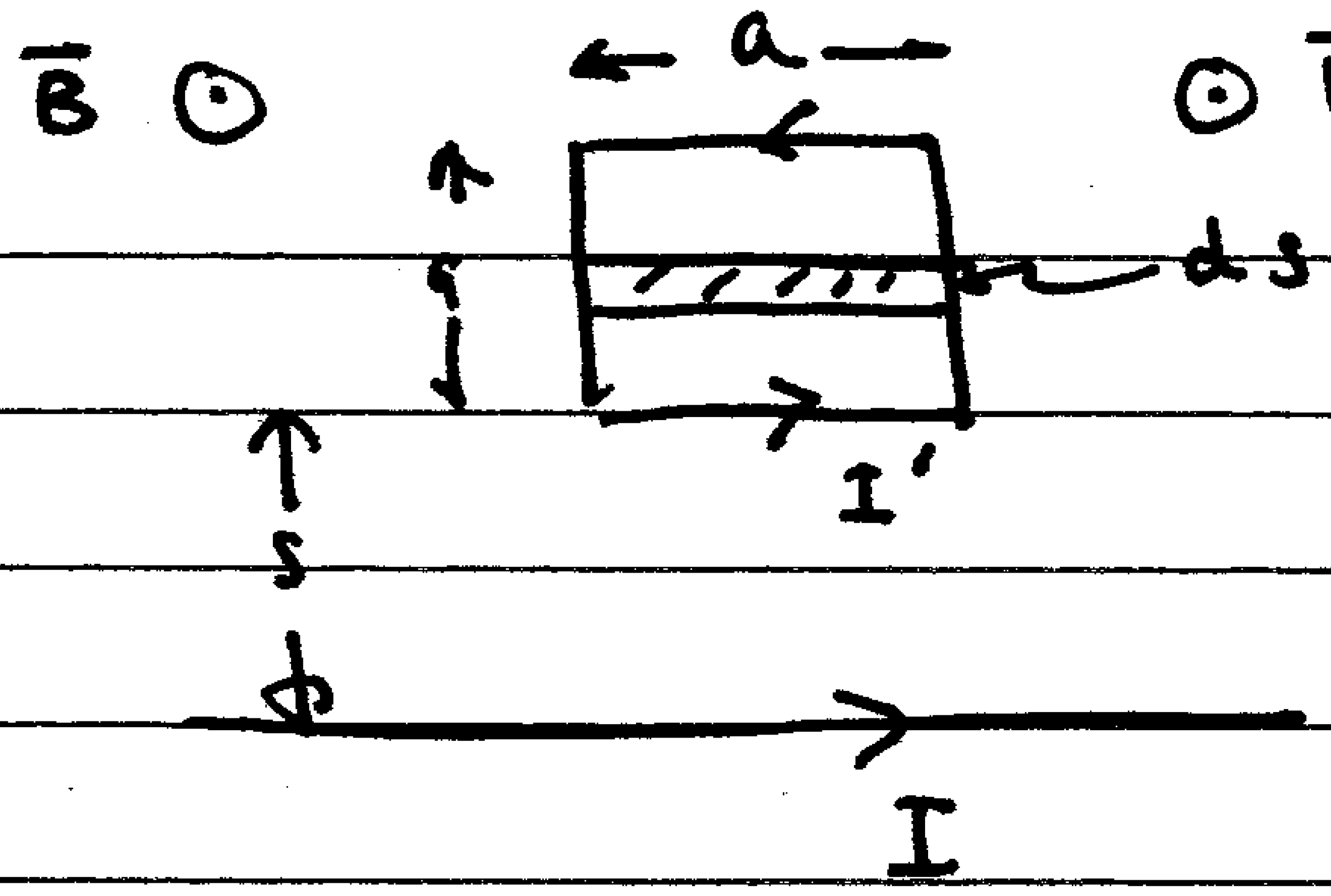
b) $\phi_B = B\pi a^2 = \mu_0 n I \pi a^2$

$$\phi_B = \mu_0 n I \pi a^2 \qquad |\mathcal{E}| = IR = \left|\frac{d\phi}{dt}\right|$$

$$R\frac{dq}{dt} = \left|\frac{d\phi}{dt}\right| \qquad q = \frac{\Delta\phi}{R}$$

$$q = \frac{\mu_0 n I \pi a^2}{R}$$

7-18)



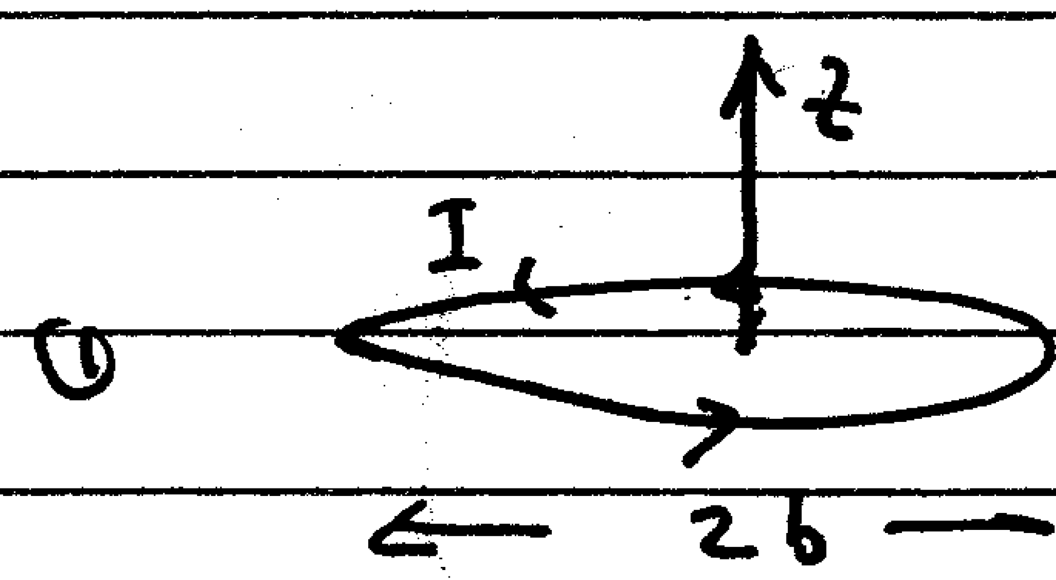
$$B = \frac{\mu_0 I}{2\pi s}$$

$$\phi = \frac{\mu_0 I}{2\pi} a \int_s^{s+a} \frac{ds'}{s'} = \frac{\mu_0 I a}{2\pi} \ln\left(\frac{s+a}{s}\right)$$

$$\mathcal{E} = \left|\frac{d\phi}{dt}\right| = I'R = \frac{dq}{dt} R$$

$$q = \frac{\Delta\phi}{R} = \frac{\mu_0 I a}{2\pi R} \ln\left(\frac{s+a}{s}\right)$$

7.22) ② radius a



a) $\vec{B}(z) = \dfrac{\mu_0 I}{2} \dfrac{b^2 \hat{z}}{(b^2+z^2)^{3/2}}$

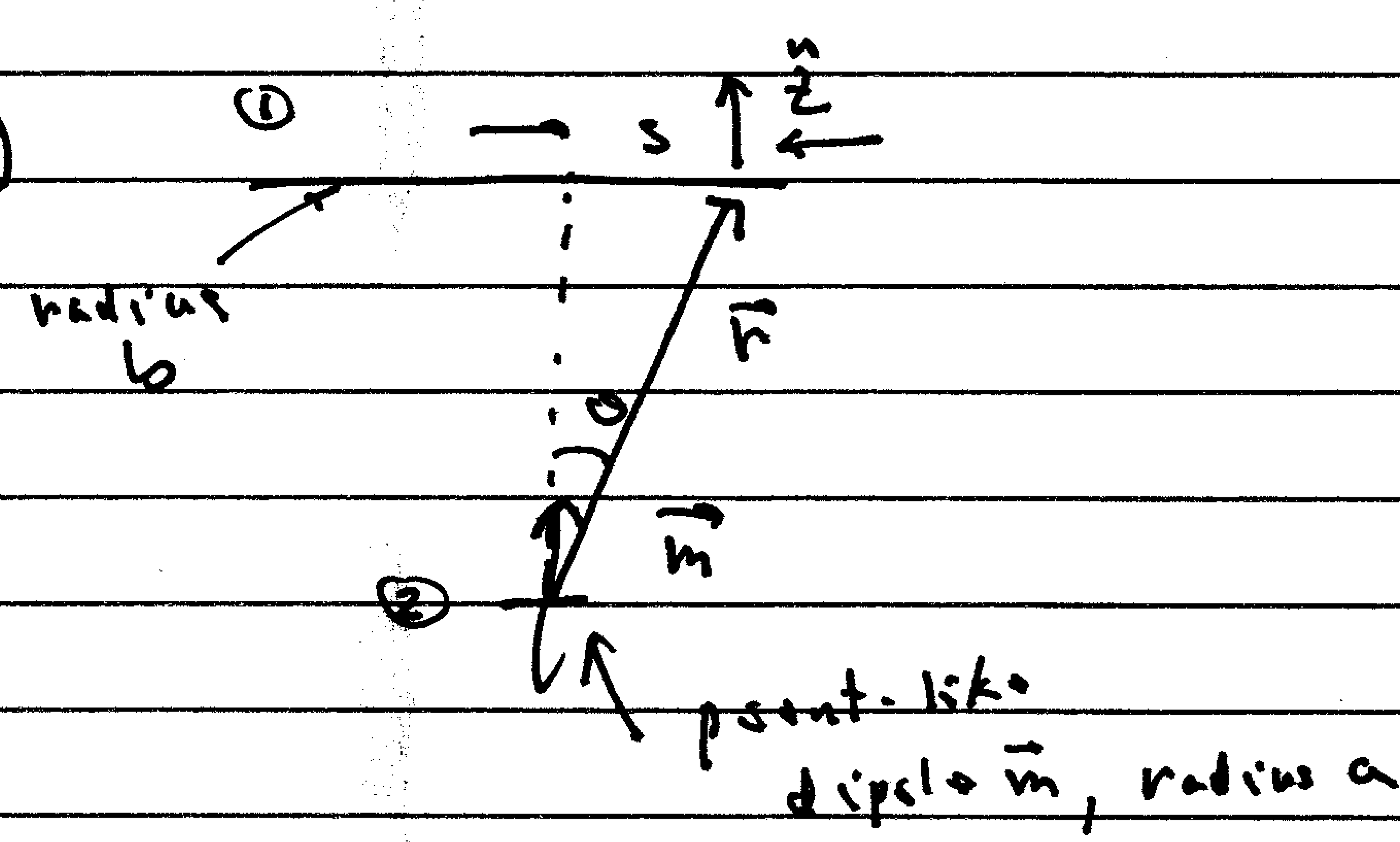
see Eqn. 5-41 on page 227.

Φ_{2-1} = flux through ② (small loop) due to ① (large loop)

$$\Phi_{2-1} = \pi a^2 B(z) = \frac{\mu_0 I \pi a^2}{2} \frac{b^2}{(b^2+z^2)^{3/2}}$$

$$M_{2-1} = \frac{\mu_0 \pi a^2 b^2}{2} \frac{1}{(b^2+z^2)^{3/2}}$$

b)



$$\vec{B} = \frac{\mu_0}{4\pi r^3}\left[3(\vec{m}\cdot\hat{r})\hat{r} - \vec{m}\right] \qquad \vec{m} = m\hat{z}$$

$$\vec{m}\cdot\hat{r} = m\cos\theta$$

ϕ_{1-2} = flux through ①, due to ②

$$d\phi_{1-2} = 2\pi s\, ds\; \vec{B}\cdot\hat{z}$$

$$\vec{B}\cdot\hat{z} = \frac{\mu_0}{4\pi r^3}\left[3(\vec{m}\cdot\hat{r})(\hat{r}\cdot\hat{z}) - \vec{m}\cdot\hat{z}\right]$$

$$= \frac{\mu_0 m}{4\pi r^3}\left(3\cos^2\theta - 1\right)$$

$$r^2 = s^2 + z^2 \qquad \cos\theta = \frac{z}{r}$$

$$3\cos^2\theta - 1 = \frac{3z^2}{r^2} - 1 = \frac{3z^2 - r^2}{r^2} = \frac{3z^2 - s^2 - z^2}{r^2}$$

$$= \frac{2z^2 - s^2}{r^2}$$

$$d\phi_{1-2} = \frac{\mu_0 m}{4\pi r^3}(2\pi s\, ds)\left(\frac{2z^2 - s^2}{r^2}\right)$$

$$\phi_{1-2} = \frac{\mu_0 m}{2}\int_0^b ds\, \frac{s(2z^2 - s^2)}{(s^2 + z^2)^{5/2}}$$

We need to do two integrals

$$I_1 = 2z^2\int_0^b ds\, \frac{s}{(s^2 + z^2)^{5/2}}$$

$$I_2 = \int_0^b ds\, \frac{s^3}{(s^2 + z^2)^{5/2}}$$

$$I_1 = 2z^2 \int_0^b ds \, \frac{s}{(s^2+z^2)^{5/2}} = 2z^2\left(-\frac{1}{3}\right)(s^2+z^2)^{-3/2}\Big|_{s=0}^{s=b}$$

$$= -\frac{2z^2}{3}\left(\frac{1}{(b^2+z^2)^{3/2}} - \frac{1}{z^3}\right)$$

$$= \frac{2}{3}\left(\frac{1}{z} - \frac{z^2}{(b^2+z^2)^{3/2}}\right)$$

$$I_2 = \int_0^b ds \, \frac{s^3}{(s^2+z^2)^{5/2}}$$

Let $s^2+z^2 = y^2$

$2s\,ds = 2y\,dy$

$s^3 ds = y(y^2-z^2)\,dy$

$$I_2 = \int_z^{(b^2+z^2)^{1/2}} dy \, \frac{y(y^2-z^2)}{y^5} = \int_z^{(b^2+z^2)^{1/2}} dy \left(\frac{1}{y^2} - \frac{z^2}{y^4}\right)$$

$$= (-1)\,y^{-1} + \frac{z^2}{3}\,y^{-3}\Big|_{y=z}^{y=(b^2+z^2)^{1/2}}$$

$$= -\frac{1}{(b^2+z^2)^{1/2}} + \frac{1}{z} + \frac{z^2}{3}\,\frac{1}{(b^2+z^2)^{3/2}} - \frac{1}{3z}$$

$$= \frac{2}{3z} + \frac{z^2}{3}\,\frac{1}{(b^2+z^2)^{3/2}} - \frac{b^2+z^2}{(b^2+z^2)^{3/2}}$$

$$= \frac{2}{3z} - \frac{b^2 + 2z^2/3}{(b^2+z^2)^{3/2}}$$

$$I_1 - I_2 = \frac{2}{3z} - \frac{2z^2/3}{(b^2+z^2)^{3/2}} - \frac{2}{3z} + \frac{b^2 + 2z^2/3}{(b^2+z^2)^{3/2}}$$

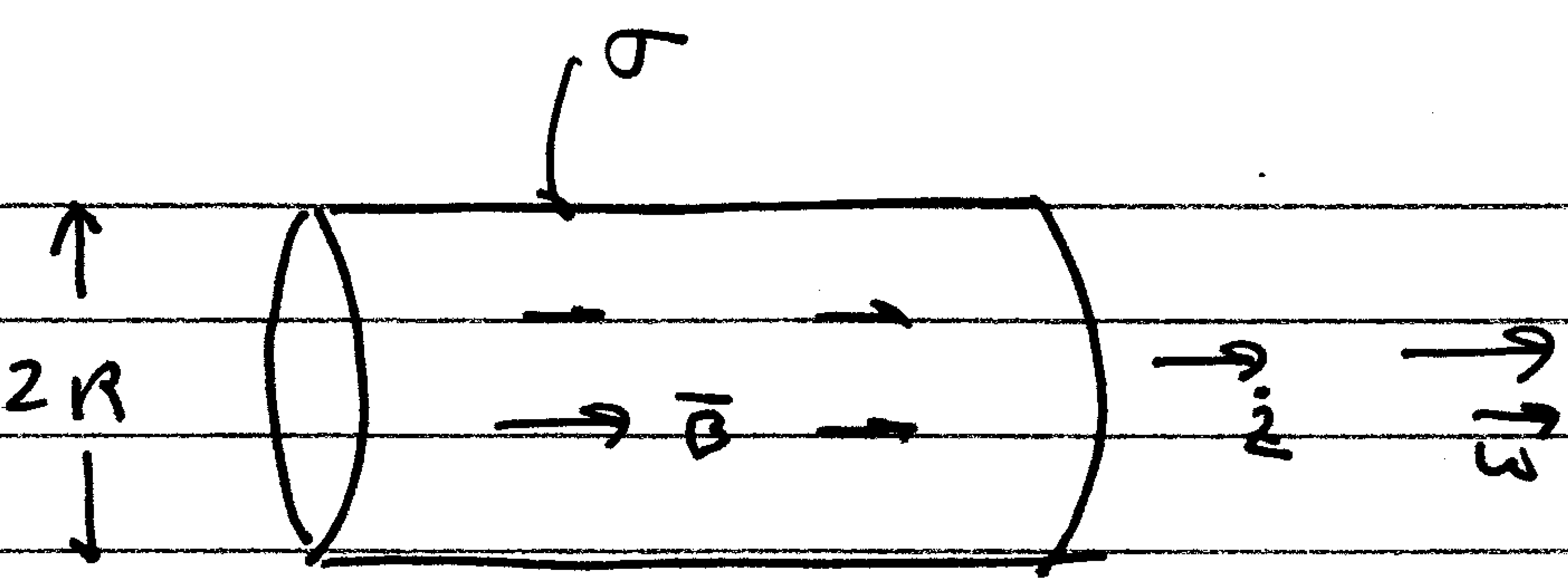
$$= \frac{b^2}{(b^2+z^2)^{3/2}}$$

$$\phi_{1-2} = \frac{\mu_0 m}{2} \frac{b^2}{(b^2+z^2)^{3/2}} \qquad m = \pi a^2 I$$

$$\phi_{12} = \frac{\mu_0 \pi a^2 b^2 I}{2} \frac{1}{(b^2+z^2)^{3/2}}$$

c) $M_{1-2} = \dfrac{\mu_0 \pi a^2 b^2}{2(b^2+z^2)^{3/2}} = M_{2-1}$

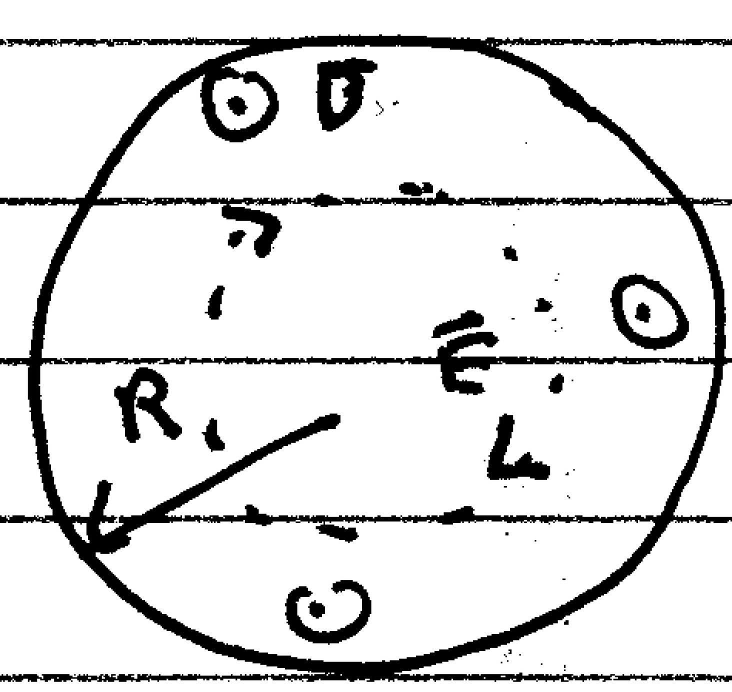
7-33)



a) For a solenoid $B = \mu_0 n I$. For this case
$nI \to K = \sigma v = \sigma \omega R$

$$\vec{B} = \mu_0 \sigma \omega R \hat{z}$$

As ω increases, B increases and an E field is created.



$$E \, 2\pi s = \frac{d}{dt}\left(B \pi s^2\right)$$

$$\vec{E} = -\frac{s}{2} \frac{dB}{dt} \hat{\varphi} \quad \text{with } \frac{dB}{dt} > 0$$

The force of E on a length l of cylinder is

$F = E(2\pi R l)\sigma$ and the torque is

$$N = E(2\pi R^2 l)\sigma \qquad E = \frac{R}{2}\frac{dB}{dt}.$$

$$E = \frac{R}{2}\mu_0 \sigma R \frac{d\omega}{dt} = \frac{\mu_0 \sigma}{2} R^2 \frac{d\omega}{dt}$$

$$N = \frac{\mu_0 \sigma^2}{2} R^2 \frac{d\omega}{dt}\left(2\pi R^2 l\right)$$

$$\vec{N} = -\pi \mu_0 l R^4 \sigma^2 \frac{d\omega}{dt} \hat{z}$$

$$\frac{dW}{dt} = N \frac{d\theta}{dt} = N\omega \qquad dW = \vec{N} \cdot \vec{\omega}\, dt$$

$$dW = -\pi \mu_0 l R^4 \sigma^2 \omega\, d\omega$$

$$W = -\frac{\pi \mu_0 l R^4 \sigma^2 \omega^2}{2}$$ This is the work

done by $\vec{E}$. the work done by an outside agent bringing the solenoid up to speed ω is

$$W = +\frac{\pi \mu_0 l R^4 \sigma^2 \omega^2}{2}$$

b) $B = \mu_0 \sigma \omega R \qquad W = \frac{1}{2} \frac{B^2}{\mu_0} (\pi R^2 l)$

$$W = \frac{1}{2\mu_0} (\mu_0 \sigma \omega R)^2 \pi R^2 l$$

$$= \frac{\pi \mu_0 \sigma^2 R^4 \omega^2 l}{2}$$

7.36) $\vec{E}(s,t) = \frac{\mu_0 I_0 \omega}{2\pi} \ln\left(\frac{a}{s}\right) \sin \omega t \, \hat{z}$

a) $\vec{J}_d = \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$

$$\vec{J}_d = \frac{\mu_0 \varepsilon_0 I_0 \omega^2}{2\pi} \ln\left(\frac{a}{s}\right) \cos \omega t \, \hat{z}$$

b) $dI_d = \vec{J}_d \cdot d\vec{a} \qquad d\vec{a} = 2\pi s \, ds \, \hat{z}$

$$I_d = \frac{\mu_0 \varepsilon_0 I_0 \omega^2}{2\pi} \cos \omega t \int_0^a 2\pi s \ln\left(\frac{a}{s}\right) ds$$

$$= \mu_0 \varepsilon_0 I_0 \omega^2 \cos \omega t \int_0^a ds \, (s \ln a - s \ln s)$$

$$\int_0^a ds \, (s \ln a - s \ln s) = \frac{s^2}{2} \ln a - \frac{s^2}{2} \ln s + \frac{s^2}{4} \Bigg|_{s=0}^{s=a}$$

$$= \frac{a^2}{2} \ln a - \frac{a^2}{2} \ln a + \frac{a^2}{4} = \frac{a^2}{4}$$

$$I_d = \frac{\mu_0 \varepsilon_0 I_0 \omega^2 a^2}{4} \cos \omega t$$

c) $I = I_0 \sin \omega t$

$$\frac{I_d}{I} = \frac{\mu_0 \varepsilon_0 \omega^2 a^2}{4} \qquad \mu_0 \varepsilon_0 = \frac{1}{c^2}$$

$$= \left(\frac{\omega a}{2c}\right)^2$$

$$\omega = \frac{2c}{a}\left(\frac{I_d}{I}\right)^{1/2} = 2\pi f$$

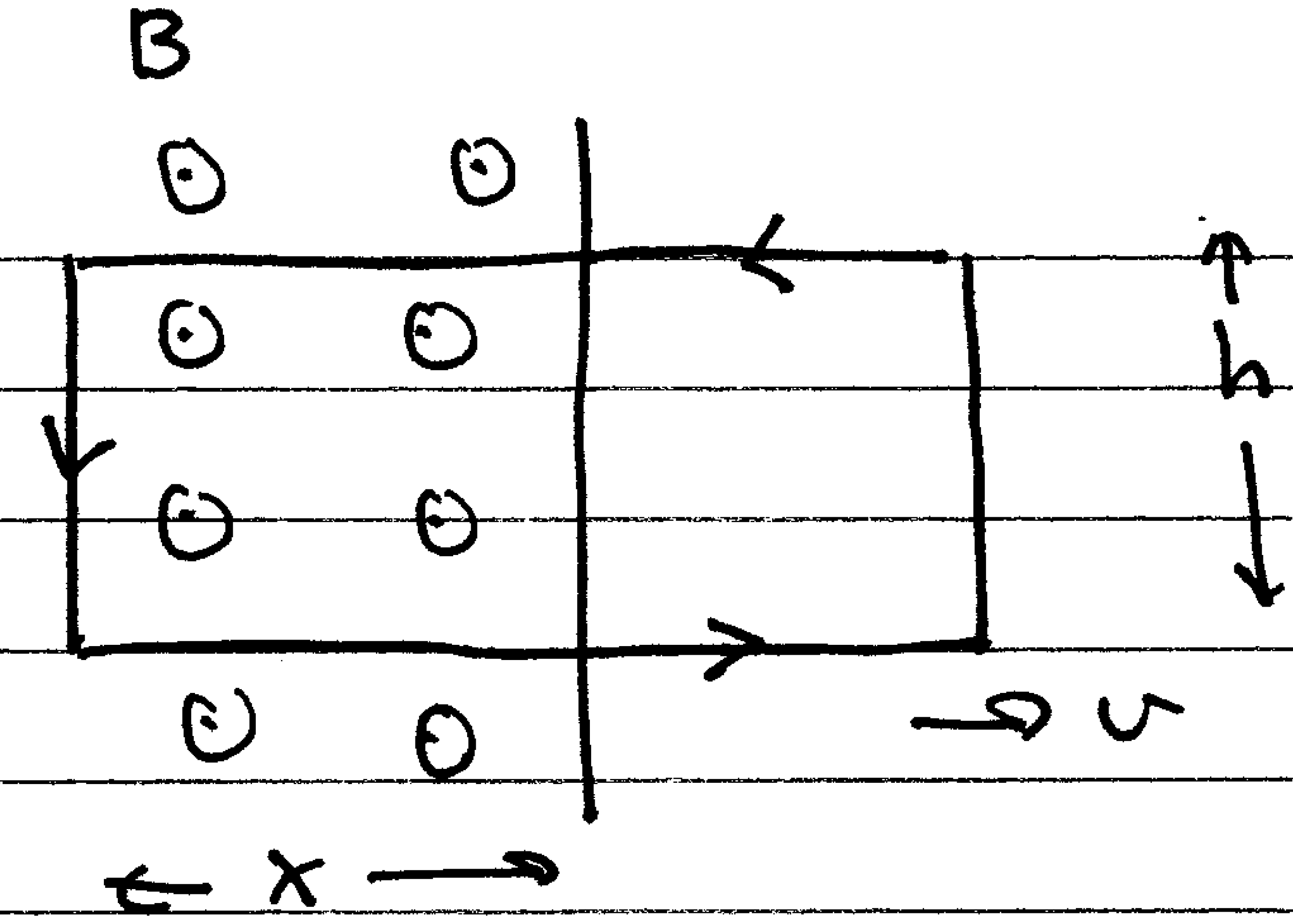
$$f = \frac{c}{\pi a}\sqrt{\frac{I_d}{I}} \qquad \text{let } \frac{I_d}{I} = 0.01 = \left(\frac{1}{10}\right)^2$$

$$a = 0.002\,m$$

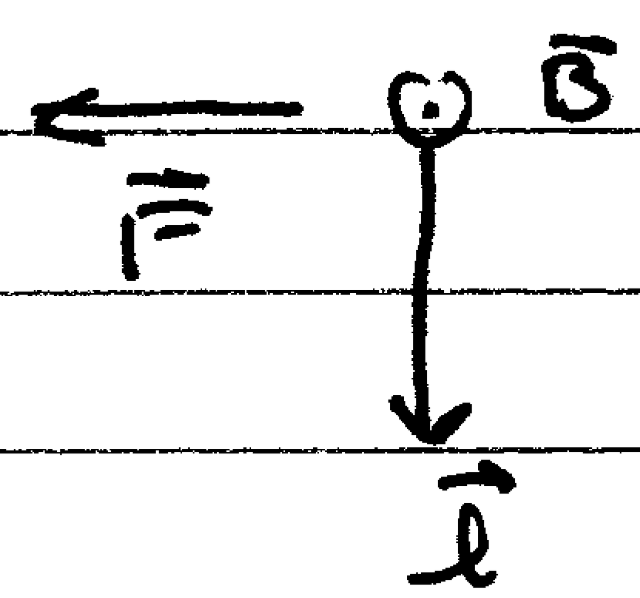
$$f = \frac{3.0 \times 10^8\, m/s}{(\pi)(0.002m)(10)}$$

$$= 4.8 \times 10^9\ Hz$$

7.55)



$$\phi = xhB \qquad \frac{d\phi}{dt} = vhB = \mathcal{E} = L\frac{dI}{dt}$$



$$\vec{F} = I\,\vec{l}\times\vec{B}$$

$$F = IhB \leftarrow$$

$$-IhB = m\frac{dv}{dt} \qquad -hB\frac{dI}{dt} = m\frac{d^2v}{dt^2}$$

$$\frac{dI}{dt} = \frac{vhB}{L} \qquad -hB\left(\frac{vhB}{L}\right) = m\frac{d^2v}{dt^2}$$

$$\frac{d^2v}{dt^2} + \frac{B^2h^2}{mL}v = 0 \qquad \omega^2 = \frac{B^2h^2}{mL}$$

this is SHO

with $\omega = \dfrac{Bh}{\sqrt{mL}}$